Bibliographie

R. E. Edwards, Fourier Series. A Modern Introduction, Vol. 1 (Graduate Texts in Mathematics, 64), xii+224 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

This is the second edition of a book appeared first in 1967. There are numerous minor corrections. In addition, the author made a few substantial changes and supplements to the exposition.

The main aim of this book is to provide an introduction to some aspects of Fourier series and related topics, in which a liberal use is made of modern techniques. It may serve as a useful preparation for Rudin's "Harmonic Analysis on Groups" and for the second volume of Hewitt and Ross' "Abstract Harmonic Analysis".

The emphasis on modern techniques effects not only the type of arguments, but also to a considerable extent the choice of material. Above all, it leads to a minimal treatment of pointwise convergence and summability. The famous treatises by Zygmund and Bary on trigonometric series cover these aspects in great detail. On the other hand, a considerable attention is paid to matters that have not yet received a detailed treatment in a book form. Among such material, there appear comments on capacity, spectral synthesis sets, Helson sets and so forth, as well as remarks on extensions of results to more general groups. Katznelson's book "Introduction to Harmonic Analysis" can be read as a companion text.

The table of contents is the following: 1. Trigonometric series and Fourier series, 2. Group structure and Fourier series, 3. Convolutions of functions, 4. Homomorphisms of convolution algebras, 5. The Dirichlet and Fejér kernels, Cesàro summability, 6. Cesàro summability of Fourier series and its consequences, 7. Some special series and their applications, 8. Fourier series in L^2, 9. Positive definite series and Bochner's theorem, 10. Pointwise convergence of Fourier series.

The reader is supposed only to be familiar with Lebesgue integration. What is needed from functional analysis (Baire's category theorem, uniform boundedness principles, the closed graph, open mapping and Hahn—Banach theorems) is dealt with in Appendices A and B. The basic terminology of linear algebra is used, but no result of any depth is assumed.

Each chapter ends with exercises, the more difficult ones being provided with hints to their solutions. The bibliography contains many suggestions for further reading. The treatment is supplemented by a list of Symbols and an Index.

The present volume is an excellent introduction. It is addressed to undergraduate students and warmly recommended to everyone who wants to make a quick acquaintance with Fourier Analysis.

F. Móricz (Szeged)

Euclidean Harmonic Analysis, Proceedings of Seminars Held at the University of Maryland, 1979, edited by J. J. Benedetto (Lecture Notes in Mathematics, 779), iv+177 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

During the spring semester of 1979 a program in Euclidean harmonic analysis was presented at the University of Maryland. This volume comprises six lecture series of them. The table of contents reads as follows.

1. L. Carleson, Some analytic problems related to statistical mechanics.

This is addressed to two main problems of classical statistical mechanics: (i) the verification of expected equilibrium thermodynamic properties, and (ii) the validity of the Gibbs theory for dynamical systems.

2. Y. Domar, On spectral synthesis in $\mathbf{R}^n$, $n \geqq 2$.

3. L. Hedberg, Spectral synthesis and stability in Sobolev spaces.

The following problem is discussed in these two lecture series: Let X be a class of distributions with support contained in a fixed subset of E of $\mathbf{R}^n$; determine whether or not a given element $\mu \in X$ is the limit in some designated topology of bounded measures contained in X. In Domar's case, the Fourier transform of X is a subset of $L^\infty(\mathbf{R}^n)$ with the weak* topology. In Hedberg's case, X is a Sobolev space with the norm topology.

4. R. Coifman and Y. Meyer, Fourier analysis of multilinear convolutions, Calderón's theorem, and analysis on Lipschitz curves.

5. R. Coifman, M. Cwikel, R. Rochberg, Y. Sagher and G. Weiss, The complex method for interpolation of operators acting on families of Banach spaces.

6. A. Córdoba, (i) Maximal functions: A problem of A. Zygmund, and (ii) Multipliers of $\mathcal{F}(L^p)$.

These three lecture series deal with the harmonic analysis of operators of L^p spaces. The problems studied have emerged mainly from the research of Zygmund, Calderón and Stein. In order to verify various L^p estimates for the Hilbert transform and related operators, R. Coifman and Y. Meyer present a range of real and complex methods. Next, G. Weiss, in a joint work with several others, set forth a theory of interpolation, which includes the Riesz—Thorin theorem and Stein's theorem for analytic families of operators. Finally, A. Córdoba solved several specific problems involving a thorough mix of many of the real methods.

The present book gives excellent accounts on the fast-growing development of Euclidean harmonic analysis, which has maintained a vital relationship with several other areas of mathematics for over 150 years. It will certainly stimulate some of the readers to attack the rather difficult problems of this important and fascinating field. We warmly recommend the book to everybody who wants to keep pace with up-to-date developments in Harmonic Analysis.

F. Móricz (Szeged)

T. W. Gamelin, Uniform Algebras and Jensen Measures (London Mathematical Society Lecture Note Series, 32), VIII+162 pages, Cambridge University Press, Cambridge—London—New York—Melbourne, 1978.

These notes are based on various courses given by the author. The unifying theme is the notion of subharmonicity with respect to a uniform algebra. Dual to the generalized subharmonic functions are the Jensen measures.

The book consists of nine chapters. Chapter 1 provides an abstract treatment of R-measures, including the basic ideas of the Choquet theory. Chapters 2 and 3 show three natural choices for R-measures: the representing measures, Arens—Singer measures and Jensen measures.

Chapter 4 is based on some unpublished work of B. Cole, in which an open Riemann surface is constructed for which the corona problem has a negative answer. Chapter 5 introduces and treats various classes of quasi-subharmonic functions, algebras generated by Hartogs series, and the abstract Dirichlet problem for function algebras. The abstract development is applied in Chapter 6 to algebras of analytic functions of several complex variables. The key to applications is a theorem of H. Bremermann asserting that the abstract subharmonic functions essentially coincide with the plurisubharmonic functions.

Chapters 7 and 8 are devoted to the theory of the conjugation operation in the setting of uniform algebras. The M. Riesz and Zygmund inequalities turn out to be valid for Jensen measures, and the constants are the same as those that arise in the case of the disc algebra. On the other hand, they fail to extend to arbitrary representing measures. In Chapter 9 the problem of characterizing the moduli of the functions in $H^2(\sigma)$ is considered. The discussion is based on Cole's proof of a theorem of Helson.

Each chapter ends with references. The book is supplemented by a List of notation and an (author and subject) Index.

The presentation is self-contained and unified. Some of the results are published here for the first time. The book may serve as a starting point for research in an area of current interest. It is highly recommended for every graduate student who wishes to continue studies in Abstract Harmonic Analysis or Functional Analysis.

F. Móricz (Szeged)

Herman H. Goldstine, A History of the Calculus of Variations from the 17th through the 19th Century (Studies in the History of Mathematics and Physical Sciences, 5), XVIII+410 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

The beginning of the calculus of variations can perhaps be dated from Fermat's elegant principle of least time, formulated in 1662 to show how a light ray was refracted at the interface between two optical media of different densities. He used the methods of the calculus to minimize the time of passage of a light ray through the two media. (By the way, Greek mathematicians were already aware of isoperimetric problems and their results were preserved for us by Pappus (c. 300 A. D.), but their methods were, of course, geometrical and not analytical).

The author attempts to trace the development of the calculus of variations during the period, in which the foundations of the modern theory were being laid. He chooses the most famous mathematicians of the period in question and concentrates on their major works.

The book is divided into seven chapters, and ends with a rich Bibliography containing about 200 items and a detailed Index.

Chapter 1 is entitled "Fermat, Newton, Leibniz, and the Bernoullis". During the 17th century mathematical notation began to improve quite markedly and the reasonable symbolisms contributed greatly to the development of mathematics. Fermat's work mentioned above seems to be clearly the first real contribution to the field. His method was adapted by John Bernoulli in 1696/97 to solve the brachystochrone problem (from *brachystos*, shortest, and *chronos*, time). The first genuine problem of the calculus of variations was formulated and solved by Newton in 1685. He investigated the motions of bodies moving through a fluid and led up to the general problem of motion in a resisting medium.

Chapters 2 and 3 ("Euler" and "Lagrange and Legendre") present the main achievements in the calculus of variations during the 18th century. In his book Euler treated 100 special problems and not only solved them but also set up the beginnings of a real general theory. His systematic investigations also served to influence the young Lagrange to seek and find a very elegant apparatus for solving problems. Lagrange explicitly formulated the famous multiplier rule, the so-called Euler—Lagrange rule, which became a sovereign tool in his hands for discussing analytical mechanics. This new tool caused Euler to name the subject appropriately the *calculus of variations*. In 1786 Legendre broke new ground by extending the calculus of variations from a study of the first variation to a study of the second variation as well.

Chapter 4 ("Jacobi and His School") is devoted to the works made in the first half of the 19th century. Legendre's analysis was not error-free, but Jacobi in 1836 wrote a remarkable paper on the second variation, in which the root of the matter was recognized. Among other things, he showed that the partial derivatives with respect to each parameter of a family of extremals satisfy the so-called Jacobi differential equation. However, none of Jacobi's results was proved in his paper. As a result a large number of commentaries were published, mainly to establish an elegant result of his on exact differentials. The celebrated Hamilton—Jacobi equation underlies some of the most profound and elegant results not only of the calculus of variations but also of mechanics, both classical and modern.

In the second half of 19th century two quite different directions were taken. On the one hand, Weierstrass went back to the first principles and not only placed the subject on a rigorous basis using the techniques of complex-variable theory, but discovered the so-called Weierstrass condition, fields of extremals, sufficient conditions for weak and strong minima, etc. On the other hand, Clebsch tentatively and A. Mayer decisively moved on quite another route. They succeeded in establishing the usual conditions for ever more general classes of problems. E.g., Mayer gave an elegant treatment of isoperimetric problems, in which he formulated his well-known reciprocity theorem. Details of their researches are presented in Chapter 5 ("Weierstrass") and Chapter 6 ("Clebsch, Mayer, and Others").

At the international mathematical congress of 1900 Hilbert gave a beautiful discussion of the calculus of variations. His greatest contributions were perhaps the discovery of his invariant integral together with the results that stem from it, the perception of the second variation as a quadratic functional with a complete set of eigenvalues and eigenfunctions, and his examination of existence theorems. Osgood, Bolza, Kneser, Carathéodory, etc., were also outstanding mathematicians at the turn of the century, whose major results are contained in Chapter 7 entitled "Hilbert, Kneser, and Others". Upon this point the present volume ends.

The above listing of the contents could hardly give a right impression of the richness of the book. It is written with a brilliant style and the text is illuminated by 66 illustrations. The book will certainly be a very instructive and profitable reading for everyone interested in the Calculus of Variations.

F. Móricz (Szeged)

G. Iooss and D. D. Joseph, Elementary stability and bifurcation theory (Undergraduate Texts in Mathematics), XV+286 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1980.

The nonlinear differential equations governing evolution problems generally contain some parameters. Therefore, the equilibrium solutions of such an equation depend on these parameters. Bifurcating sulutions are equilibrium solutions which form intersecting branches in a suitable space of functions. One of the central problems in bifurcation theory is: how do stability properties of equilibrium solutions change at bifurcation points?

The book is a very good text for teaching the principles of bifurcation. It gives a general theory abstracted from the detailed theory required for particular applications, and providing the reader with a "skeleton on which detailed structures of the applications must rest."

The following types of equilibrium are treated: steady solutions of autonomous problems, periodic solutions of nonautonomous problems, periodic solutions of steady problems, subharmonic solutions of periodic problems, subharmonic bifurcating solutions of periodic solutions of autonomous problems. Bifurcation of periodic solutions of autonomous and nonautonomous problems into "asymptotically quasi-periodic" solution is considered as well.

The book follows the simplest way of teaching the subject, starting with the analysis of one and two-dimensional problems and later demonstrating how the lower-dimensional problems relate to high-dimensional problems. Instead of the Center Manifold Theorem, the Implicit Function Theorem and the Fredholm Alternative are used for the computation of power series solutions and for the determination of qualitative properties of the bifurcating solutions.

Owing to its simplicity and generality, the book should be very useful to persons working in fields as diverse as biology, chemistry, engeneering, mathematics, and physics.

L. Hatvani (Szeged)

J. E. Marsden and M. McCracken, The Hopf bifurcation and its applications (Applied Mathematical Sciences, 19), XIII+408 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1976.

The Hopf bifurcation occurs in connection with dynamical systems containing some parameters and refers to the development of periodic orbits ("self-oscillations") from a stable fixed point, as a parameter crosses a critical value. This phenomenon can be illustrated by the following example. A rigid, hollow sphere with a small ball inside hangs from the ceiling and rotates about a vertical axis through its center. For small rotation frequences the bottom of the sphere is a stable point. But if the frequency exceeds a critical value then this equilibrium becomes unstable, the ball moves up the side of the sphere to a new fixed point. For each value of the frequency greater than the critical one there is a stable, invariant circle of fixed points.

The applications necessitate examination of Hopf bifurcation for vector fields and diffeomorphisms given on manifolds. The book originated at a seminar given in Berkeley in 1973—74 and contains contributions of many authors. It offers an excellent discussion of the theoretical results and applications of this topic. The basic tool is the "Center Manifold Theorem" which enables the infinite-dimensional problems to be reduced to finite dimensional ones. The authors give a survey on the necessary preliminaries from functional analysis, thus their book is readable for a wide circle of readers interested in this theory and its applications.

The book treats not only the new directions of research but also the classical results. For example, a translation of Hopf's original and generally unavailable paper is included. In Hopf's original approach, the determination of the stability of the resulting periodic orbits is, in concrete problems, an unpleasant calculation. The authors give explicit algorithms for this calculation which are easy to apply in examples. The method of averaging also is used for reducing the problem and establishing stability properties.

Chapters are devoted to partial differential equations, where the key assumption is that the semi-flow defined by the equations is smooth in all variables for $t>0$.

The importance of bifurcation theory is in its very close connections with applications. The reader can find interesting problems arising in fluid dynamics, population dynamics, celluar biology etc.

To sum up, we can warmly recommend this book for mathematicians, users of mathematics as well as science students.

L. Hatvani—J. Terjéki (Szeged)

R.O. Wells, Differential Analysis on Complex Manifolds (Graduate Texts in Mathematics, 65), x+260 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

This book is the second edition of a succesful work which was first published by Prentice-Hall, Inc. (1973). The main program of the author is to give a very elegant development of Hodge's theory of harmonic integrals and Kodaira's characterization of projective algebraic manifolds.

The first four chapters discuss four somewhat different areas of mathematics.

Firstly differentiable manifolds and vector bundles are studied. Besides summarizing some of the basic definitions and results, this chapter contains some nontrivial embedding theorems, the continuous and C^∞ classification of vector bundles. Almost-complex structures and calculus of differentiable forms are also introduced.

Roughly speaking, sheaf theory gives techniques for passage from local information to global information. This theory is desribed in chapter 2.

Chapter 3 is an exposition of the basic ideas of Hermitian differential geometry with applications to Chern classes and holomorphic line bundles. The general theory of elliptic differential operators on compact differentiable manifolds can be found in the following chapter. The decomposition theorem of Hodge is proved here, asserting that for a self-adjoint differential operator the vector space of the sections is the orthogonal direct sum of the finite-dimensional null space and of the range of the operator. The Hodge's representation of the de Rahm cohomology by harmonic forms is also described.

The following chapter 5 is a main chapter of the book. Compact complex manifolds are studied here with the application of the previous discussions. Many basic theorems of this field are proved, for example the Lefschetz decomposition theorem, the Hodge decomposition theorem, Hodge's generalization of the Riemannian period relations for integrals of harmonic forms on Kähler manifolds, the Kadaira—Spencer upper semicontinuity theorem, etc. This chapter contains also a new section in addition to the first edition of the book. This is the classical finite dimensional representation theory for $sl(2C)$ which is then used for giving a natural proof of the Lefschetz decomposition theorem.

In the last chapter the famous Kodaire Embedding Theorem is proved, which asserts that a compact complex manifold admits an algebraic embedding into a complex projective space iff it is a Hodge manifold.

The book should be suitable for a graduate level course on the general topic of complex manifolds. The text is relatively self-contained but assumes familiarity with the usual first year graduate courses.

Z. I. Szabó (Szeged)

H. Werner und R. Schaback, Praktische Mathematik II (Methoden der Analysis), Hochschultext; Zweite, neubearbeitete und erweiterte Auflage, VIII+388 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1979.

The aim of this textbook is to provide a rigorous background of certain results widely used in Numerical Analysis. The treatment is self-contained, it requires the knowledge of calculus only.

The present volume consists of four chapters. Chapter 1 treats the theory of interpolation, involving multiple dimensional interpolation and fast Fourier transform. Chapter 2 is devoted to approximation theory, among others, to the Remes algorithm, the Fourier and Čebyšev expansions of continuous functions. Chapter 3 begins with spline functions, including cubic splines, B-splines etc. These results are then applied to the problem of representation of linear functionals, in particular, to numerical differentiation and integration. Chapter 4 deals with numerical methods for the initial value problem of ordinary differential equations. Both one-step methods, especially the classical Runge—Kutta methods, and predictor-corrector methods are presented in details. The notions of consistency, stability and convergence of a method plays central role in the treatment. This chapter ends with the presentation of stability theorems of Dahlquist.

Throughout the text there are various examples and figures (altogether 36) illuminating the material presented, and giving hints to further results found in the literature. The orientation of the reader is helped by a notational index as well as an author and subject index. Among the references one finds references to more than 40 textbooks.

The material presented in this well-readable book belongs to the main body of up-to-date Numerical Analysis. It will certainly be useful as a textbook for both science and engineering undergraduate students.

F. Móricz (Szeged)

A. Weron (ed.), Probability on Vector Spaces II, Proceedings, Błażejewko, Poland, 1979. (Lecture Notes in Mathematics, 628), XIII+324 pages, Springer-Verlag, Berlin—Heidelberg—New York, 1980.

From the editor's foreword: "This volume contains 30 contributions — the written and often extended versions of most lectures given at the Conference. A great majority of papers present new results in the field and the rest are expository in nature. The material in this volume complements the material in the earliner volume Probability Theory on Vector Spaces, Proceedings Lecture Notes in Math. vol. 656, 1978, Springer-Verlag".

Lajos Horváth (Szeged)

George W. Whitehead, Elements of Homotopy Theory (Graduate Texts in Mathematics, 61), XXI+744 pages, Springer-Verlag, New York—Heidelberg—Berlin, 1979.

Homotopy theory is one of the most essential field of topology, which had its inception in the work of L. E. J. Brouwer. The book is concerned with the basic ideas and results of this theory in a modern treatment.

The fundamental notions and problems of the theory such as homotopy classes of mappings, fibrations, CW-complexes, the H- and H'-spaces, the Hurewitz map of homotopy group into homology group etc. are introduced in the first four chapters. The Hurewitz theorem is also proved, asserting that the Hurewitz homomorphism is an isomorphism if the basic space is $(n-1)$-connected.

The fifth chapter is devoted to the study of CW-approximations of spaces and of the extension problem of maps from a relative CW-complex onto the CW-complex. In the following chapter a new homology group is introduced, with the help of which results parallel to those of obstruction theory can then be proved.

The relationships among the homotopy groups of spaces arising from a fibration are expressed by an exact sequence. But the behaviour of the homology groups is much more complicated, and this can be examined only in certain cases. These problems are discussed in chapter 7, while the following chapter is devoted to the study of several cohomology operations.

For a 0-connected space X and positive integer N, one can embed X in a space X^N such that (X^N, X) is a $(N+1)$ -connected relative CW-complex with $\pi_q(X^N)=0$ for all $q>N$. The space X^{N+1} can be constructed from X^N with the help of a certain cohomology class $k^{N+1}\in H^{N+2}(X^N, \pi_{N+1}(X))$, and X is determined up to weak homotopy type by the so-called Postnikov system $\{X^N, k^{N+2}\}$ of X. In Chapter 9 the Postnikov systems are used to give an alternative treatment of obstruction theory for maps into X.

In the last three chapters the author turns to the detailed study of H-spaces, homotopy operations and homology theories without the dimension axiom.

The book is a very careful and clear work. It is a very good introduction to the field, at the same time it can be considered as a high level survey of the subject. It is assumed that the reader is familar with fundamental group theory and singular homology theory, including the universal coefficients and Künneth theorems.

Z.I. Szabó (Szeged)